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Giant magnetoresistance in hybrid superconductor/ferromagnetic sandwich heterostructures

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Abstract

The perpendicular electron transport through a ferromagnetic bilayer (F_1/F_2) in contact with a superconductor (S) was investigated theoretically. The conductance is calculated in the Kubo formalism with Green functions found as the solutions of the Gorkov equations. It is shown that the giant magnetoresistance (GMR) defined as the relative difference in conductivity between the parallel and antiparallel alignments of the magnetizations in F_1 and F_2 behaves differently in the ballistic and diffusive regimes. In the former case, the GMR amplitude can be fairly large, whereas in the latter case, it almost vanishes. The interpretation of this behaviour is given by comparing the contributions to the total resistance of the Andreev reflection at the F_1/S interface and the usual quantum reflection at the F_1/F_2 interface.

1. Introduction

The problem of spin-dependent electron transport in nanoheterostructures has attracted a lot of attention during the last decade. In heterostructures consisting of successive ferromagnetic thin layers separated by paramagnetic layers, the so-called giant magnetoresistance (GMR) was discovered [1]. Another type, for which the spin polarization of the current affects the measured resistance, are the hybrid heterostructures: ferromagnetic metal/superconductor (F/S) bilayers or multilayers. The theoretical descriptions of the spin-polarized electron transport for the two types of heterostructure have common features. Namely, it is considered that the characteristics of the current carriers in a ferromagnetic metal are different for different spin directions. So for ballistic transport one has to take into account the spin asymmetry of the electron band

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structure (e.g. exchange splitting), whereas for diffusive transport the most important factor is the spin dependence of the elastic mean free paths of electrons. In GMR structures, when the magnetizations of the successive ferromagnetic layers are antiparallel (AP), an electron with definite spin (no spin-flip processes) moves successively through layers of high and low conductivities, suffering reflections at the interfaces. In contrast, in the case of parallel magnetizations (P), electrons with favourable spin for high conductivity move freely through the structure leading to a 'short circuit' (see the review [2]).

The situation for F/S structures is similar to the case of antiparallel orientation of magnetizations in GMR structures. In F/S structures one has to take into account Andreev reflection [3] (A.R.), when electrons with definite spin reflect at an F/S interface as holes with opposite spin, leaving the current-driving Cooper pair in the superconductor. For ballistic transport, only the fraction of electrons in the majority-spin subband are concerned in the A.R., and the remaining fraction are reflected at the F/S interface like at a usual potential barrier, so these electrons cannot contribute to the current. For diffuse transport the current in the ferromagnet is carried by electron–hole pairs with different spins, created in the process of A.R. As a result the resistivity of the F/S structure is for both regimes higher than that of the single F layer. The theory of ballistic transport for F/S structure was developed in [4] and in more detail in [5] where a list of references can be found. The case of diffuse transport in F/S structures was considered in [6, 7].

Now the question arises of whether the electron transport in the F/S structure is determined by processes similar to those in GMR structures with antiparallel orientation of magnetizations in the adjacent ferromagnetic layers: does the resistivity of the hybrid GMR structure in contact with a superconductor depend on the orientation of the magnetization or does it reach its maximum value due to the presence of superconductivity suppressing the GMR? This problem was approached in [8, 9]. In [8] the motion of electrons through the spin-valve structure consisting of two ferromagnetic layers (or domains) with parallel and antiparallel directions of magnetizations in contact with a superconductor was described by the diffusion equations with different diffusion constants for up- and down-spin electrons. It was shown that due to the A.R. the resistivities of one- and two-domain structures are equal and so the GMR is completely suppressed. The authors considered that GMR would be restored if one took into account spin-flip processes. A different model was used in [9]. The multilayered structure $[F/N]_n S$, where F(Co) is the ferromagnetic layer, N(Cu) is the paramagnetic metal layer, S is the superconductor, was described in terms of an spd tight-binding Hamiltonian with parameters fitted for an *ab initio* band calculation. In the case considered in [9], the conductance of the system was calculated through the Landauer formula generalized in [10] for the case of A.R. from the superconductor. The results of the numerical calculation of the conductance of the system for the P and AP cases for a given set of the parameters (only the thickness of the P(Cu) layer or the number n of periods was a variable parameter) have shown that the GMR value is zero within the accuracy of the computation. These results seem to be rather puzzling, since in experiments [11, 12] superconducting contacts were most often used to measure the CPP-GMR (CPP standing for current perpendicular to the plane). In all of these experiments rather high values of the GMR for $[F/N]_n$ multilayers was found. To resolve this contradiction, the authors of [13] have calculated the dependence of the GMR on the strength of the spin–orbit interaction in the F layers for a model similar to the one used in [9]. The numerical simulation has shown that with increasing spin–orbit interaction ξ, which mixes the up- and down-spin channels, the value of the GMR increases also up to a value of $\sim$100% at $\xi \approx 0.08$ eV. For larger ξ, it decreases, vanishing at $\xi \approx 0.17$ eV. It is interesting to note that for values of $\xi \geqslant 0.08$ eV, the GMR for the system in contact with a superconductor in the normal state coincides with that for one in contact with a superconductor in the superconducting state.

Of course, the suggested mechanism for restoring the GMR due to spin–orbit interaction has to be taken into account for thicknesses of F layers comparable to the spin-diffusion length (SDL—this length is determined at low temperatures by the spin–orbit contribution to the scattering potential). Detailed analyses [14] of the experimental data on CPP-GMR for a lot of multilayers of different types have shown that for layer thicknesses much smaller than the SDL, the values of the observed GMR are fairly large despite the fact that Nb superconducting contacts were used. (We recall that in [13] it was predicted that the GMR would tend to zero if spin-mixing processes were non-effective.) One more important conclusion of [14] was that the GMR cannot be explained by just extrinsic effects (scattering by imperfections); it is necessary to also take into account intrinsic effects, coming from electron reflections at perfect interfaces. The latter effects are more pronounced in the case of ballistic transport. The ballistic regime of transport in a GMR sandwich in contact with a superconductor was investigated in [15]. It was shown that the GMR oscillates as a function of the ferromagnetic layer thickness around a non-zero average value, crossing the zero value at several points, so in general the GMR is not suppressed in the ballistic regime. Simultaneously, in [15] the conclusion of the paper [8], that the GMR is zero for a diffuse regime, was rederived.

In this paper we present the results of an investigation of the GMR for one- and two-domain ferromagnetic layers in contact with a superconductor, taking into account the spin-dependent bulk scattering of electrons, the barrier between domains and A.R. at the ferromagnetic/superconductor interface.

2. Model

We consider a spin-valve sandwich of the type F/F/S, where the Fs are ferromagnetic layers (domains) with thicknesses a and b, the magnetizations of which can be oriented parallel or antiparallel to each other, and S is a superconducting layer. A simple two-band (spin-up and spin-down) free-electron model is adopted for this calculation. So the Hamiltonian of the system is written as

$$H = H_F + H_S \tag{1a}$$

$$H_F = \sum_{\sigma=+(\downarrow),-(\uparrow)} \int_{r\in F} \left[\left(\frac{\hat{p}^2}{2m} - \varepsilon_F + \operatorname{sgn}(\sigma)\varepsilon_{\text{ex}} \right) \times \psi_\sigma^{\text{s}*}(r)\psi_\sigma^{\text{s}}(r) + \gamma_{\text{sd}}(r)\left(\psi_\sigma^{\text{s}*}(r)\psi_\sigma^{\text{d}}(r) + \text{h.c.}\right) \right] \mathrm{d}^3 r \tag{1b}$$

$$H_S = \int_{r\in S} \left[\sum_\sigma \left(\frac{\hat{p}^2}{2m} - \varepsilon_F \right) \psi_\sigma^{\text{s}*}(r)\psi_\sigma^{\text{s}}(r) + \left(\Delta(r)\psi_\uparrow^{\text{s}*}(r)\psi_\downarrow^{\text{s}*}(r) + \text{h.c.}\right) \right] \mathrm{d}^3 r \tag{1c}$$

where σ is the electron spin index,

$$\varepsilon_{\text{ex}} = \frac{k_F^{\uparrow 2} - k_F^{\downarrow 2}}{2m}$$

is the exchange energy, ε_F, k_F^σ are Fermi energy and momentum respectively. The second term in (1b) describes the scattering of quasi-free s electrons into almost localized d states. In bulk ferromagnetic metals d states may make a contribution to the current [16], but as we consider that there are no d states in the superconductor, d electrons are completely reflected at F/S interfaces and will not make any contribution to the current. However, s–d scattering in ferromagnetic dirty d-metal alloys remains the most important mechanism of s-electron scattering for the case under consideration. Further, we will consider the random s–d scattering potential γ_{sd} to be much smaller than ε_F and we will calculate the mean free path in the Born approximation. Δ is the order parameter of the superconductor. Now it is easy to write down the system of Gorkov equations for the normal (G) and anomalous (F) Green functions:

$$\left[\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial z^2}-\varkappa^2\right)+\varepsilon_F+\varepsilon_{\mathrm{ex}}-\gamma_{\mathrm{sd}}^2\mathcal{G}_{\mathrm{dd}}^{\uparrow\uparrow}(z,z)\right]\mathcal{G}_{\mathrm{ss}}^{\uparrow\uparrow}(z,z')+\Delta F_{\mathrm{ss}}^{\uparrow\downarrow}(z,z')=\delta(z-z') \tag{2a}$$

$$\Delta^*\mathcal{G}_{\mathrm{ss}}^{\uparrow\downarrow}(z,z')-\left[\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial z^2}-\varkappa^2\right)+\varepsilon_F-\varepsilon_{\mathrm{ex}}+\gamma_{\mathrm{sd}}^2\mathcal{G}_{\mathrm{dd}}^{\downarrow\downarrow}(z,z)\right]F_{\mathrm{ss}}^{\uparrow\downarrow}(z,z')=0. \tag{2b}$$

The system of equations (2) is written in the $(\varkappa, z)$ representation, where $\varkappa$ is the electron momentum projection on the XY-plane and z is the axis perpendicular to the plane of the layer. We consider that the system is infinite and homogeneous in the XY-plane. The terms $\varepsilon_{\mathrm{ex}}$ and $\gamma^2\mathcal{G}_{\mathrm{dd}}^{\uparrow\uparrow}$ are different from zero and $\Delta = 0$ if z belongs to the F layer, and vice versa in S layers. The system (2) is written for the layer with its magnetization parallel to the $\uparrow$-spin. For the layer with the opposite direction of magnetization, $\varepsilon_{\mathrm{ex}}$ has to be changed to $-\varepsilon_{\mathrm{ex}}$ and $\mathcal{G}_{\mathrm{dd}}^{\downarrow\downarrow\,(\uparrow\uparrow)}$ to $\mathcal{G}_{\mathrm{dd}}^{\uparrow\uparrow\,(\downarrow\downarrow)}$. The main difference of system (2) from that usually employed for F/S structures (see, for example, [17]) is that we took into account s–d scattering. Otherwise the term $\gamma_{\mathrm{sd}}^2\mathcal{G}_{\mathrm{dd}}$ in (2) has to be replaced by $\gamma_{\mathrm{ss}}^2\mathcal{G}_{\mathrm{ss}}$ and the additional term $\gamma_{\mathrm{ss}}^2 F_{\mathrm{ss}}$ will appear. In this case the system of equations (2) becomes non-linear and so too complicated. In our case the function $\mathcal{G}_{\mathrm{dd}}^{\sigma\sigma}(z,z)$ may be considered as a constant along the z-direction if it is averaged over wavelength oscillations, and the system of equations (2) may then be solved analytically. The explicit expressions for the Green function are given in appendix A.

The current j^σ for each direction of electron spin may be written as

$$j^\sigma=\int[G^{\sigma\sigma}(z,z')E_{\mathrm{eff}}^{\sigma}(z')+G^{\sigma,-\sigma}(z,z')E_{\mathrm{eff}}^{-\sigma}(z')]\,\mathrm{d}z' \tag{3}$$

where the conductances $G^{\sigma\sigma'}$ are given by the generalized Fisher–Lee formula [18]:

$$G^{\sigma\sigma'}(z,z')=\frac{4e^2}{\pi\hbar}\left(\frac{\hbar^2}{2m}\right)^2\sum_{\varkappa}A_{\varkappa}^{\sigma\sigma'}(z,z')\overleftrightarrow{\nabla}_z\overleftrightarrow{\nabla}_{z'}A_{\varkappa}^{\sigma'\sigma}(z',z) \tag{4}$$

where

$$\overleftrightarrow{\nabla}_z=(1/2)(\overrightarrow{\nabla}_z-\overleftarrow{\nabla}_z)$$

is the antisymmetric gradient operator and

$$A_{\varkappa}^{\sigma\sigma'}=(\mathrm{i}/2)(\mathcal{G}_{\varkappa}^{\sigma\sigma'}-\mathcal{G}_{\varkappa}^{\sigma'\sigma}).$$

Here $\mathcal{G}_{\varkappa}^{\sigma\sigma}$ is the conventional Green function and $\mathcal{G}_{\varkappa}^{\sigma,-\sigma}=F_{\varkappa}^{\sigma,-\sigma}$ is the anomalous Green function different from zero only in the presence of Cooper pairs (superconductor). $E^\sigma(z')$ is the effective electrical field acting on the carrier (electron or hole). Here it is important to notice that the expression (3) for the conductance is written in the 'bubble' approximation, and following the procedure suggested in [19], the vertex corrections are taken into account by introducing effective spin-dependent electrical fields $E^\sigma(z)\equiv\partial\mu^\sigma(z)/\partial z$, where $\mu^\sigma(z)$ is the electrochemical potential. These fields have to be found from the condition of non-divergency of each spin-channel current (we consider the case of no spin mixing):

$$\frac{\partial j^\sigma(z)}{\partial z}=0. \tag{5}$$

Later we will show that to fulfil conditions (5) the effective fields $E^\sigma(z)$ have to be different for different spin directions and in each layer ($E^\sigma(z)$ is a step-like function), and also that due to the charge and spin accumulation, a finite spin-dependent voltage drop occurs at each interface (F/F and F/S).

3. Ballistic regime

If the thicknesses of all layers are much smaller than the electron mean free paths, one can neglect the electron scattering. If one substitutes into equation (4) the expressions for the Green functions (see appendix A) in the limit of $\gamma_{\rm sd} \to 0$, the conductance does not depend on the coordinates z and z'. In this ballistic limit the condition of non-divergency of the current is fulfilled automatically and the expressions for the conductances of the system for parallel (G_P) and antiparallel (G_{AP}) orientation of the magnetizations can be written in the following forms:

$$G_P = \frac{4e^2}{\pi\hbar}\sum_{\varkappa}(1 - R^2) \tag{6a}$$

$$G_{AP} = \frac{4e^2}{\pi\hbar}\sum_{\varkappa}\frac{(1 - R^2)(1 - r^2)}{\rm Den} \tag{6b}$$

$$\begin{aligned}{\rm Den} = {} & 1 + 2r^2R^2 + r^4 - 2Rr(\cos 2c^{\uparrow}a - \cos 2c^{\downarrow}a)(1 + r^2) \\ & - 2r^2\left[R^2 \cos 2(c^{\uparrow} + c^{\downarrow})a - \cos 2(c^{\uparrow} - c^{\downarrow})a\right]\end{aligned} \tag{6c}$$

where

$$r = \frac{c^{\uparrow} - c^{\downarrow}}{c^{\uparrow} - c^{\downarrow}}$$

represents the effective spin polarization,

$$R = \frac{c^{\uparrow}c^{\downarrow} - c_2^2}{c^{\uparrow}c^{\downarrow} + c_2^2}$$

$$c^{\uparrow(\downarrow)} = \sqrt{(k_F^{\uparrow(\downarrow)})^2 - \varkappa^2} \qquad c_2 = \sqrt{(k_F^S)^2 - \varkappa^2}.$$

Also, a is the thickness of the intermediate ferromagnetic layer, k_F^S is the Fermi wave vector in the superconducting layer. The upper limit of the sum over $\varkappa$ is equal to the minimum value of $k_F^{\uparrow(\downarrow)}$ or k_F^S. The expression (6a) for G_P coincides in the appropriate limit with the expression for the conductance of the single ferromagnetic layer in contact with a superconductor obtained, for example, in [5]. It is important to notice that if $k_F^{\uparrow}$, $k_F^{\downarrow}$ and k_F^S do not coincide and so in general $R^2 \neq 0$, the conductance of the system decreases compared to the case of the absence of superconducting contact. The expression (6b) contains in the numerator two factors: $(1 - R^2)$ and $(1 - r^2)$. The first one is specific to Andreev reflection and the second one describes the usual reflection of polarized electrons at the F/F interface. So if only these two factors are taken into account, considering the denominator equal to unity, the conductance of the AP configuration is smaller than the one for the P configuration, and the GMR survives. Let us consider then the effect of the denominator in expression (6b). It describes the multiple reflections of an electron which moves inside the ferromagnetic layer adjacent to the superconductor, as in a Fabry–Perot interferometer. These multiple reflections are responsible for the formation of quantum well states within the layer. As a result, the conductance G_{AP} is an oscillatory function of the arguments $k_F^{\uparrow(\downarrow)}a$, $(k_F^{\uparrow} \pm k_F^{\downarrow})a$, but it never diverges or becomes negative. A similar behaviour of the conductance was predicted in [4], for a structure composed of a ferromagnetic layer sandwiched between a superconducting contact on one side and a thin oxide barrier on the other side.

Now we come to the question of whether G_P is always larger than G_{AP} or whether, for some values of the parameters, G_{AP} can be equal to or even larger than G_P. For very small polarization $r \ll 1$, from expressions (6) it is easy to obtain the following approximate expression for the GMR:

$$\mathrm{GMR} = \frac{\sigma^P - \sigma^{AP}}{\sigma^{AP}} = 2\sum_{\varkappa} r^2 R^2 = 2\sum_{\varkappa}\left(\frac{c^\uparrow - c^\downarrow}{c^\uparrow + c^\downarrow}\right)^2\left(\frac{c^\uparrow c^\downarrow - c_2^2}{c^\uparrow c^\downarrow + c_2^2}\right)^2. \tag{7}$$

This expression is definitely positive. Without any superconducting contact, the GMR would be given by

$$\mathrm{GMR} = 2\sum_{\varkappa} r^2.$$

It is interesting to note that expression (7) coincides with the expression for the MR in a spin-valve tunnel junction [20], after substitution for c_2 of the modulus of the imaginary electron momentum inside the barrier. The physics is similar for the two phenomena: in both cases the electrons undergo reflections at the interface: F/I (I: insulator) or F/S. These spin-dependent reflections change the spin-dependent density of states in the ferromagnet near the interface and, correspondingly, change the polarization of the current.

For larger r, the conductances G^P and G^{AP} are plotted in figure 1 versus the square of the effective polarization r^2. The following parameters were used: $k_F^\uparrow = 1$ Å^{-1}, $k_F^S = 1.3$ Å^{-1} and $a = 5c_0$ ($c_0 = 4.06$ Å is the lattice parameter of Co for hcp structure). $a = 5c_0$ corresponds to ten atomic monolayers. $k_F^\downarrow$ was varied from 1 to 0, so correspondingly r^2 was changing from 0 to 1. As can be seen in figure 1, the conductances for both magnetic configurations decrease as r^2 increases. G^{AP} exhibits also some weak oscillations as a function of r^2, but it remains smaller than G^P for almost the whole range of r^2.

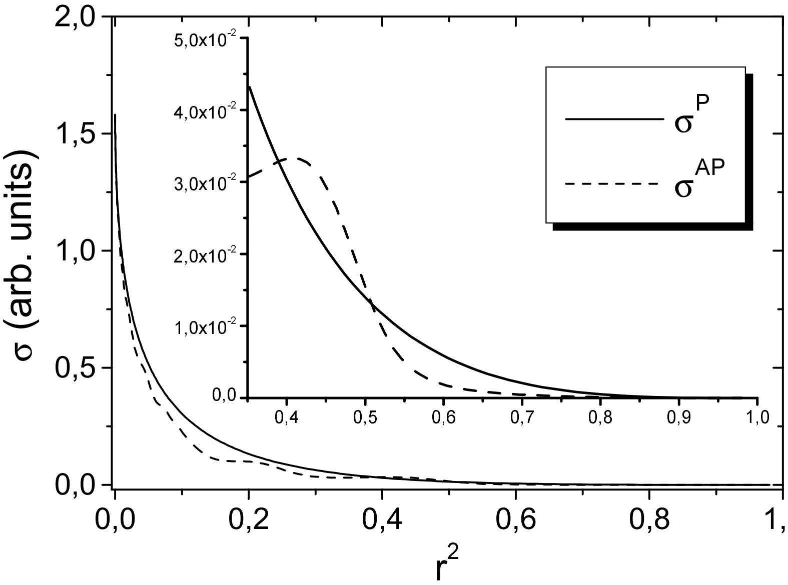

Figure 1. The dependences of the conductivities for the parallel and antiparallel configurations as functions of the effective spin polarization r^2. The parameters are $k_F^\uparrow = 1$ Å^{-1}, $k_F^S = 1.3$ Å^{-1}, $a = 20.3$ Å, $l^\uparrow = l^\downarrow = 10\,000$ Å (quasi-ballistic regime).

In figure 2, the GMR is plotted versus the thickness a for a given value of $r^2 = 0.16$. Figure 2 shows that the GMR $= (G^P - G^{AP})/G^{AP}$ oscillates around a non-zero positive value and for the ballistic regime tends to the asymptotic limit 40% for $a > 100$ Å. Of

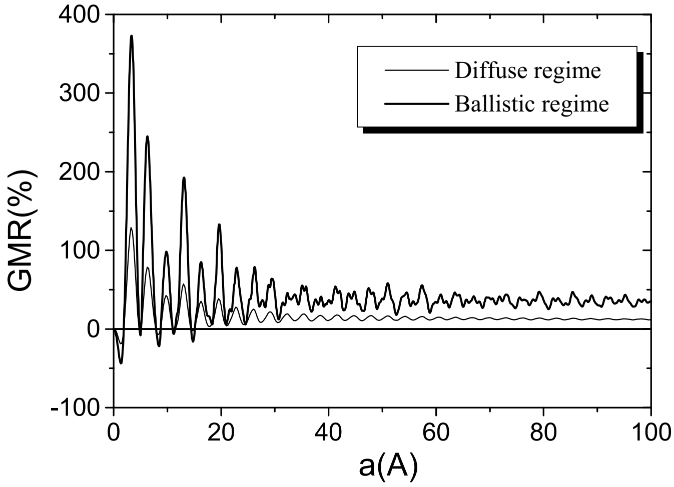

Figure 2. The GMR for the ballistic and diffusive regimes as functions of the thickness of the ferromagnetic layers ($a = b$). The parameters are $k_F^{\uparrow} = 1$ Å^{-1}, $k_F^{\downarrow} = 0.429$ Å^{-1}, $k_F^S = 1.3$ Å^{-1}. For the ballistic regime: $l^{\uparrow} = l^{\downarrow} = 10\,000$ Å; for the diffuse regime: $l^{\uparrow} = 120$ Å, $l^{\downarrow} = 40$ Å.

course, the thickness of the layer can change only in steps equal to the lattice parameter $c_0/2$. It is interesting to note that the situation is similar to the case of a spin-valve tunnel junction with a paramagnetic metal layer inserted between one ferromagnetic electrode and an insulating barrier [21–23]. In this case, it was shown that the paramagnetic layer (for instance Cu inserted between Co and Al_2O_3) can constitute a spin-dependent quantum well. Oscillations in the tunnel magnetoresistance (TMR) were predicted for such a system as a function of the paramagnetic layer thickness with a period given by the Fermi wavelength in this layer. However, a crucial difference between this case and the present one is that here, the GMR oscillates around a finite positive value, whereas in a tunnel junction, the TMR oscillates around zero. Consequently, for tunnel junctions, averaging over a distribution of paramagnetic layer thickness caused by roughness and/or increasing the paramagnetic layer thickness leads to a strong decay in the TMR amplitude. In contrast, in the present case, averaging over a distribution of thickness and/or increasing the thickness of the ferromagnetic layers leads to a non-zero GMR amplitude which depends on the values of r^2 and R^2.

In figure 3 the dependence of the GMR on the effective spin polarization r^2 is plotted for the thickness a equal to ten monolayers of Co. We have also calculated the a-dependence of the conductivity for other values of r^2. These dependences are similar to the one shown in figure 2. The resonances of G^{AP} exhibit rather sharp peaks at $a = 2\pi n/k_F^{\downarrow}$ if $1 - r^2 \ll 1$, i.e. $k_F^{\downarrow} \to 0$. In this case, the system becomes a real Fabry–Perot interferometer for electrons.

4. Diffuse regime

Now we suppose that it is possible to neglect exchange splitting of the s-type-carrier (electrons and holes) bands, but their up and down mean free paths in the ferromagnet are different due to s–d scattering. The detailed theory of the GMR for this model was presented in [24] and is often used for the interpretation of CPP-GMR experiments. In this case the formal expressions for the currents $j_{\uparrow}^{P(AP)}$ for P and AP configurations through the system, for example, may be

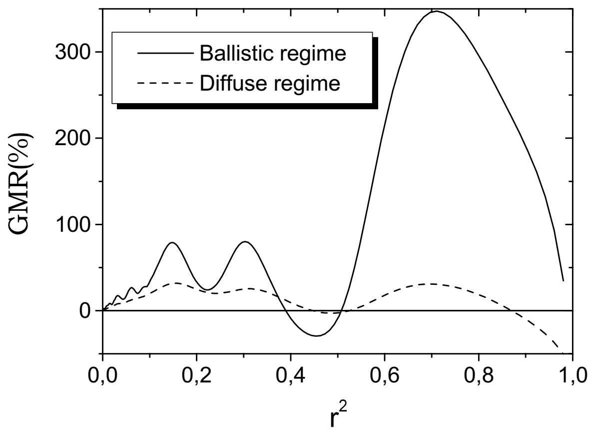

Figure 3. The GMR as a function of the square of the effective spin polarization r^2 for the ballistic and diffuse regimes for $a = b = 20.3$ Å. The parameters are $k_F^{\uparrow} = 1$ Å^{-1}, $k_F^S = 1.3$ Å^{-1}. For the ballistic regime: $l^{\uparrow} = l^{\downarrow} = 10\,000$ Å; for the diffuse regime: $l^{\uparrow} = 120$ Å, $l^{\downarrow} = 40$ Å.

written as

$$j_1^{\uparrow AP} = E_1^{\uparrow} \int \frac{1}{d_1} \varkappa \, \mathrm{d}\varkappa + \frac{1}{2} \int \Bigg[\frac{E_1^{\uparrow}}{d_1} - \frac{E_2^{\uparrow}}{d_2} (1 + \mathrm{e}^{-2d_2 a}) + \frac{E_2^{\downarrow}}{d_1} \mathrm{e}^{-2d_2 a} (1 - \mathrm{e}^{-2d_1 a}) + \frac{E_1^{\downarrow}}{d_2} \mathrm{e}^{-2(d_1+d_2)a} + \Delta v_1^{\uparrow} + \Delta v_1^{\downarrow} \, \mathrm{e}^{-2(d_1+d_2)a} + (\Delta v_2^{\uparrow} + \Delta v_2^{\downarrow}) \mathrm{e}^{-2d_2 a} \Bigg] \mathrm{e}^{2d_1(z-z_1)} \varkappa \, \mathrm{d}\varkappa \tag{8a}$$

$$j_2^{\uparrow AP} = E_2^{\uparrow} \int \frac{1}{d_2} \varkappa \, \mathrm{d}\varkappa - \frac{1}{2} \int \Bigg[\frac{E_2^{\uparrow}}{d_2} \mathrm{e}^{-2d_2 a} - \frac{E_2^{\downarrow}}{d_1} \mathrm{e}^{-2d_2 a} (1 - \mathrm{e}^{-2d_1 a}) - \frac{E_1^{\downarrow}}{d_2} \mathrm{e}^{-2(d_1+d_2)a} \, \Delta v_1^{\downarrow} \, \mathrm{e}^{-2(d_1+d_2)a} - (\Delta v_2^{\uparrow} + \Delta v_2^{\downarrow}) \mathrm{e}^{-2d_2 a} \Bigg] \mathrm{e}^{2d_2(z-z_1)} \varkappa \, \mathrm{d}\varkappa + \frac{1}{2} \int \Bigg[\frac{E_1^{\uparrow}}{d_1} - \frac{E_2^{\uparrow}}{d_2} + \Delta v_1^{\uparrow} \Bigg] \mathrm{e}^{-2d_2(z-z_1)} \varkappa \, \mathrm{d}\varkappa \tag{8b}$$

$$j^{\uparrow P} = E_P^{\uparrow} \int \frac{1}{d_1} \varkappa \, \mathrm{d}\varkappa - \int \Bigg[\frac{E_P^{\uparrow}}{d_1} - \frac{E_P^{\downarrow}}{d_2} - (\Delta v_{2P}^{\uparrow} + \Delta v_{2P}^{\downarrow}) \Bigg] \mathrm{e}^{2d_1(z-z_2)} \varkappa \, \mathrm{d}\varkappa \tag{8c}$$

and the bias voltage is

$$v = E_1^{\sigma} b + E_2^{\sigma} a + \Delta v_1^{\sigma} + \Delta v_2^{\sigma} = E_P^{\sigma}(a+b) + \Delta v_{2P}^{\sigma}$$

where

$$d_{1\,(2)} = \mathrm{Im} \sqrt{(k_F^{\uparrow\,(\downarrow)})^2 - \varkappa^2 + \mathrm{i}\, 2k_F^{\uparrow\,(\downarrow)}/l^{\uparrow\,(\downarrow)}}.$$

Here, $l^{\uparrow\,(\downarrow)}$ are the spin-dependent mean free paths, E_P^{σ} and $E_{1,2}^{\sigma}$ are the effective electrical fields acting on the carriers with spin σ for the P and AP orientations in the first and second ferromagnetic layer, $\Delta v_{1,2}^{\sigma}$ and Δv_{2P}^{σ} are the spin-dependent voltage drops at F/F and F/S interfaces due to the charge and spin accumulation. The unknown values of E^{σ}, E_P^{σ}, $\Delta v_{1,2}^{\sigma}$ and Δv_{2P}^{σ} have to be found from the condition of non-divergency of the currents: $\partial j_{\sigma}^{P\,(AP)}/\partial z = 0$; e.g. the sum of the terms in (4) proportional to the exponents $\mathrm{e}^{-d_1 z}$ and $\mathrm{e}^{-d_2 z}$ has to vanish and

$j_1^{\sigma AP} = j_2^{\sigma AP}$. The system of equations is then easily solved and the resistances for P and AP orientations are equal:

$$R^P = R^{AP} = \frac{v}{j^\uparrow + j^\downarrow} = \frac{(a+b)(\rho^\uparrow + \rho^\downarrow)}{4} \tag{9}$$

where $\rho^{\uparrow(\downarrow)}$ are the resistivities for $\uparrow$ ($\downarrow$) spin channels for the bulk ferromagnet. Expression (9) coincides with expression (11) from [8] if one neglects the spin-flip scattering. It is worthwhile to notice that we used the Kubo formalism and the authors of [8] employed the diffusion equation.

According to (9), on the assumption that the GMR originates from spin-dependent scattering rates in the magnetic materials, we find that there is no GMR effect in the presence of superconducting contact. This conclusion coincides with the results obtained in [8].

The absence of GMR in this case can be qualitatively understood as follows. In a ferromagnetic metal, currents for up- and down-spin electrons are not equal. However, in a BCS superconductor, the current is driven by spinless Cooper pairs, so up- and down-spin currents are equivalent. To maintain this equivalence, electrons undergo Andreev reflection at the ferromagnet/superconductor interface and spin accumulation appears at this interface. Due to this accumulation, a jump Δv of chemical potentials (of different signs for up- and down-spin electrons) arises. The values of these jumps are

$$\Delta v^\uparrow = v\frac{\rho^\downarrow - \rho^\uparrow}{\rho^\uparrow + \rho^\downarrow} = -\Delta v^\downarrow$$

for the P configuration and

$$\Delta v^\uparrow = -\Delta v^\downarrow = v\frac{(a-b)(\rho^\downarrow - \rho^\uparrow)}{(a+b)(\rho^\uparrow + \rho^\downarrow)}$$

for the AP configuration, where v is the voltage drop across the total structure. In particular, $\Delta v_\uparrow = \Delta v_\downarrow = 0$ for $a = b$ for the AP configuration. Then, the additional drop of voltage in the parallel configuration exactly equalizes the resistances for the P and AP configurations, so the GMR is suppressed.

5. General case

Now let us consider the model where both mechanisms, the electron's reflection at the exchange barrier between the layers with antiparallel magnetization and spin-dependent elastic scattering of electrons within the ferromagnetic layers, influence the electron transport. In this case we have to substitute the expressions for Green functions given in appendix A in the expression (4) for the conductance, and substitute this expression into the right-hand side of equation (3), using the condition $\partial j^\sigma/\partial z = 0$ to find the system of equations and defining all effective fields and voltage drops at interfaces F/F and F/S in a way similar to the one used for the diffuse regime. The expressions for currents from which we can get the system of equations for fields and voltage drops are given in appendix B. To demonstrate the influence of spin-dependent scattering on the GMR of the system considered, the dependence of the GMR on the thickness of the ferromagnetic layers is shown in figure 2 for the case of a finite electron mean free path: $l^\uparrow = 120$ Å and $l^\downarrow = 40$ Å. For the case $a/l < 1$ (figure 2) the curves for finite and infinite (ballistic regime) mean free paths exhibit similar oscillating behaviours due to the presence of quantum well states within the intermediate layer in the AP configuration. The scattering decreases the amplitude of these oscillations but the GMR values are still high enough to be measured in experiments. For the case $a/l > 1$ (figure 4) oscillations of the GMR are damped

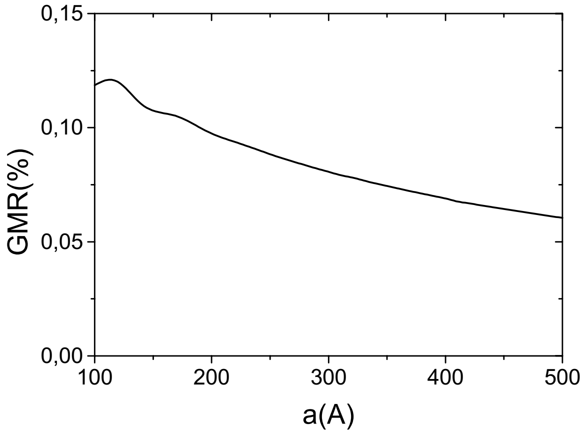

Figure 4. The GMR as a function of the thickness of the ferromagnetic layers $a = b$ for the diffuse regime. The parameters are the same as for figure 1.

and the GMR value gradually decreases, almost vanishing at $a/l > 10$. This suppression of the GMR may be explained if one compares ballistic and diffusive regimes. As we have shown, for the first case the GMR arises due to the additional reflection at the F/F interface for AP orientation, and for the latter case the GMR is completely suppressed. In the case under consideration, the electron transport is influenced by both factors: reflection and bulk scattering. Naturally for layer thickness larger than the mean free paths, the total resistance of the structure is governed mainly by the bulk scattering rather than by reflection at the F/F interface. So the electron transport has mostly diffuse character, but it was shown that in this case superconducting contact suppresses the GMR. A similar influence of scattering on the GMR value is illustrated in figure 3. Comparing curves, one can see that scattering decreases the value of the GMR and even changes its sign for large values of r^2.

6. Conclusions

In [14] the authors (see [14] and references therein) have measured CPP-GMR for spin-valve multilayers composed from various ferromagnetic 3d metals and alloys. They found that the values of the GMR for different metals vary from several per cent up to $\sim$100%. Usually the thicknesses of the ferromagnetic layers do not exceed 100 Å, so the electron transport through them may be considered as quasi-ballistic. The values calculated in our approach are high enough in the range of F thicknesses 100–500 Å (figure 4). So our results at least do not contradict this series of experiments. However, in [9, 13] another scenario was suggested for the GMR in structures with superconducting contacts. As numerical simulation of the GMR in such structures [9] gives a zero value of the GMR, in [13] the spin-mixing processes due to the spin–orbit interaction were taken into account and it was shown that the GMR increases with the parameter ξ of the spin–orbit interaction up to the value $\xi \approx 0.08$ eV and decreases after that point. If we return to the experimental data, it was found that multilayers with Co [11, 12] exhibit a GMR value several times larger than the one with permalloy [25], and estimation of the spin-diffusion lengths l_{sd} gives $l_{\mathrm{sd}}(\mathrm{Co}) \approx 590$ Å [26] $\gg l_{\mathrm{sd}}(\mathrm{Py}) \approx 33$–$53$ Å [25, 27]. So, the amplitude of the spin–orbit interaction in Co is much smaller than that in Py and, following the consideration of [13], the GMR for Co multilayers has to be smaller than that for Py ones, which contradicts the experimental data.

It is interesting to analyse qualitatively the experimental data on the GMR dependence of Py/Cu/Py sandwiches on the thickness of Py presented in [25]. It was observed that the GMR value decreases from 11% for the thickness of Py equal to 150 Å to 8% for thickness equal to 300 Å. This decrease was explained in [25] by the influence of the spin-mixing processes, due to the spin–orbit scattering. The fit of the experimental points by the curve predicted by the theory [24] gave an unexpectedly small value of the spin-diffusion length $l_{sd} = 55$ Å. But we should mention that in the theory [24] the influence of the superconducting contact on the GMR was not taken into account. The theory presented above qualitatively describes this decrease of the GMR (see figure 4), without taking into account spin-flip processes. The underlying mechanisms of this decrease are completely different from that suggested in [24]; namely, with increase of the Py thickness, the electron transport in the system undergoes a crossover from the ballistic regime (non-zero GMR) to the diffuse one (zero GMR). We think that a complete interpretation of the experiments requires taking into account both mechanisms: the electron's reflection at F/F interfaces suggested in this paper and the spin mixing suggested in [13], especially in the case of Py multilayers.

Until now we have neglected the intraband ss scattering, considering the ss scattering rate $1/\tau_{ss}$ to be much smaller than the s–d one $1/\tau_{sd}$—that is, $\tau_{ss}^{-1}/\tau_{sd}^{-1} \sim \rho^{s}/\rho^{d} \ll 1$ where $\rho^{s\,(d)}$ is the s (d) density of states at the Fermi level.

To show that ss scattering does not qualitatively change the results obtained above, we have to underline that, as follows from the comparison of the ballistic and diffuse regimes, electron scattering leads to decrease of the amplitude of the oscillations of the conductivity and the GMR versus the layer thickness, and the characteristic damping is proportional to

$$d_1 + d_2 = 1/l^{\uparrow} + 1/l^{\downarrow}$$

(see the expression for 'den' in appendix B). As was shown, for example in [28], for the single exchange-split s-band model, the characteristic damping length for the anomalous Green function $F(z)$ is equal to

$$\xi_{ss} = \sqrt{\frac{4 l_{ss} v_F \hbar}{3\varepsilon_{ss}^{exch}}}$$

where l_{ss} is the electron's elastic mean free path, due to ss scattering, v_F is the Fermi velocity, ε_{ss}^{exch} is the exchange splitting of the s band. So we can neglect ss scattering if

$$\left(\frac{1}{l_{sd}^{\uparrow}} + \frac{1}{l_{sd}^{\downarrow}}\right)^{-1} \Big/ \xi_{ss} \sim \sqrt{(\rho_s/\rho_d)\frac{3\varepsilon_{ss}^{exch}}{4\hbar v_F (1/l_{sd}^{\uparrow} + 1/l_{sd}^{\downarrow})}} \ll 1.$$

If we put in this expression the values of the parameters characteristic for 3d ferromagnetic metals, this ratio is equal to 0.1–0.2.

Appendix A. Green functions

For the AP case:

$$\mathcal{G}_{11}^{\uparrow\uparrow} = \frac{1}{2\mathrm{i}k_1}\left(\mathrm{e}^{\mathrm{i}k_2|z-z_1|} + \frac{\mathcal{A}^{\uparrow}\mathrm{e}^{\mathrm{i}k_2 a} - r\mathcal{B}^{\uparrow}\mathrm{e}^{-\mathrm{i}k_2 a}}{\tilde{\mathrm{den}}}\mathrm{e}^{-\mathrm{i}k_1(z+z'-2z_1)}\right) \tag{A.1}$$

$$\mathcal{G}_{22}^{\uparrow\uparrow} = \frac{1}{2\mathrm{i}k_2\,\tilde{\mathrm{den}}}(\mathcal{B}^{\uparrow}\mathrm{e}^{\mathrm{i}k_2(z-z_2)} - \mathcal{A}^{\uparrow}\mathrm{e}^{-\mathrm{i}k_2(z-z_2)})(r\mathrm{e}^{\mathrm{i}k_2(z'-z_1)} - \mathrm{e}^{-\mathrm{i}k_2(z'-z_1)}) \tag{A.2}$$

$$F_{11}^{\downarrow\uparrow} = \frac{(1-r^2)(1-R)}{2k_3\,\tilde{\mathrm{den}}}\mathrm{e}^{\mathrm{i}k_2^*(z-z_1)}\mathrm{e}^{-\mathrm{i}k_1(z'-z_1)} \tag{A.3}$$

$$F_{22}^{\downarrow\uparrow} = -\frac{1-R}{2k_3\,\tilde{\mathrm{den}}}(\mathrm{e}^{\mathrm{i}k_1^*(z-z_1)} + r\mathrm{e}^{-\mathrm{i}k_1^*(z-z_1)})(r\mathrm{e}^{\mathrm{i}k_2(z'-z_1)} - \mathrm{e}^{-\mathrm{i}k_1(z'-z_1)}) \tag{A.4}$$

$$\mathcal{G}_{21}^{\uparrow\uparrow} = -\frac{\mathrm{e}^{-\mathrm{i}k_1(z'-z_1)}}{\mathrm{i}(k_1+k_2)\tilde{\mathrm{den}}}(\mathcal{B}^{\uparrow}\mathrm{e}^{\mathrm{i}k_2(z-z_2)} - \mathcal{A}^{\uparrow}\mathrm{e}^{-\mathrm{i}k_2(z-z_2)}) \tag{A.5}$$

$$F_{21}^{\downarrow\uparrow} = \frac{k_2}{k_3}\frac{(1-R)\mathrm{e}^{-\mathrm{i}k_1(z'-z_1)}}{(k_1+k_2)\tilde{\mathrm{den}}}(\mathrm{e}^{\mathrm{i}k_1^*(z-z_1)} + r\mathrm{e}^{-\mathrm{i}k_1^*(z-z_1)}) \tag{A.6}$$

where

$$\begin{aligned}\mathcal{A}^{\uparrow} &= \mathrm{e}^{\mathrm{i}k_1^*a}R - \mathrm{e}^{-\mathrm{i}k_1^*a}r\\ \mathcal{B}^{\uparrow} &= -\mathrm{e}^{\mathrm{i}k_1^*a} - \mathrm{e}^{-\mathrm{i}k_1^*a}rR\\ \tilde{\mathrm{den}} &= r\mathcal{A}^{\uparrow}\mathrm{e}^{\mathrm{i}k_2a} - \mathcal{B}^{\uparrow}\mathrm{e}^{-\mathrm{i}k_2a}.\end{aligned}$$

For the P case:

$$\mathcal{G}^{\uparrow} = \frac{\mathrm{e}^{\mathrm{i}k_1|z-z'|}}{2\mathrm{i}k_1} + \frac{\mathrm{e}^{-\mathrm{i}k_1(z+z'-2z_2)}}{2\mathrm{i}k_1}R \tag{A.7}$$

$$F^{\downarrow\uparrow} = -\frac{1-R}{2k_3}\mathrm{e}^{\mathrm{i}k_2^*(z-z_2)}\mathrm{e}^{-\mathrm{i}k_1(z'-z_1)}. \tag{A.8}$$

Here z_1, z_2 are the coordinates of the F_1/F_2 and F_2/S interfaces.

Appendix B. Expressions for current

$$j^{\sigma} = \int j_{\varkappa}^{\sigma}\varkappa\,\mathrm{d}\varkappa \tag{B.1}$$

$$\begin{aligned}
j_{1\varkappa}^{\uparrow} = \frac{E_1^{\uparrow}}{d_1} &- \frac{\mathrm{e}^{-2d_1(z-z_1)}}{2}\left(\frac{E_1^{\uparrow}}{d_1} - \Delta v_0^{\uparrow}\right)\mathrm{e}^{-2d_1b}\\
&- \frac{\mathrm{e}^{2d_1(z-z_1)}}{2}\left\{\frac{E_1^{\uparrow}}{d_1}\left(2 - \frac{(1-r^2)(\mathcal{C}^{\uparrow} - \mathcal{D}^{\uparrow}\mathrm{e}^{-4d_2a})}{\mathrm{den}}\right)\right.\\
&- \frac{E_2^{\uparrow}}{d_2\,\mathrm{den}}\left[\mathcal{C}^{\uparrow}(1-\mathrm{e}^{-2d_2a}) - \mathcal{D}^{\uparrow}(\mathrm{e}^{-2d_2a} - \mathrm{e}^{-4d_2a})\right]\\
&+ \frac{E_1^{\downarrow}}{d_2\,\mathrm{den}}(1-r^2)(1-R^2)\mathrm{e}^{-2(d_1+d_2)a}\\
&- \frac{E_2^{\downarrow}}{d_1\,\mathrm{den}}\left[\mathrm{e}^{-2(d_1+d_2)a}(1+r^2) - \mathrm{e}^{-2d_2a} - r^2\mathrm{e}^{-2(2d_1+d_2)a}\right]\\
&+ \Delta v_1^{\downarrow}\frac{(1-r^2)^2(1-R^2)}{\mathrm{den}}\,\mathrm{e}^{-2(d_1+d_2)a)} + \frac{\Delta v_1^{\uparrow}}{\mathrm{den}}(1-R^2)(\mathcal{C}^{\uparrow} - \mathcal{D}^{\uparrow}\mathrm{e}^{-4d_2a})\\
&+ \frac{\Delta v_2^{\uparrow} + \Delta v_2^{\downarrow}}{\mathrm{den}}(1-r^2)(1-R^2)(\mathrm{e}^{-2d_2a} - r^2\mathrm{e}^{-2(2d_1+d_2)a})\\
&+ \left(1 - \frac{(1-r^2)(\mathcal{C}^{\uparrow} - \mathcal{D}^{\uparrow}\mathrm{e}^{-4d_2a})}{\mathrm{den}}\right)\left(\frac{E_1^{\uparrow}}{d_1} - \Delta v_0^{\uparrow}\right)\mathrm{e}^{-2d_1b}\\
&\left.- \frac{(1-r^2)^2(1-R^2)}{\mathrm{den}}\left(\frac{E_1^{\downarrow}}{d_2} - \Delta v_0^{\downarrow}\right)\mathrm{e}^{-2d_2b}\right\}
\end{aligned} \tag{B.2}$$

$$
\begin{aligned}
j_{2\varkappa}^{\uparrow} = {} & \frac{E_2^{\uparrow}}{d_2\,\mathrm{den}}(\mathcal{C}^{\uparrow} - r^2\mathcal{D}^{\uparrow}\mathrm{e}^{-4d_2a}) - \frac{\mathrm{e}^{2d_2(z-z_1)}}{2\,\mathrm{den}}\left\{\frac{E_1^{\uparrow}}{d_1}\mathcal{D}^{\uparrow}\mathrm{e}^{-4d_2a}\right. \\
& - \frac{E_1^{\downarrow}}{d_2}(1-r^2)(1-R^2)\mathrm{e}^{-2(d_1+d_2)a} \\
& + \frac{E_2^{\uparrow}}{d_2}\left[\mathcal{C}^{\uparrow}\mathrm{e}^{-2d_2a} + \mathcal{D}^{\uparrow}(\mathrm{e}^{-2d_2a} - (1+r^2)\mathrm{e}^{-4d_2a})\right] \\
& + \frac{E_2^{\downarrow}}{d_1}(1-R^2)\left[\mathrm{e}^{-2(d_1+d_2)a} - \mathrm{e}^{-2d_2a} + r^2\mathrm{e}^{-2(d_1+d_2)a}(1-\mathrm{e}^{-2d_1a})\right] \\
& + \Delta v_1^{\uparrow}(1-r^2)\mathcal{D}^{\uparrow}\mathrm{e}^{-4d_2a} - \Delta v_1^{\downarrow}(1-r^2)(1-R^2)\mathrm{e}^{-2(d_1+d_2)a} \\
& + (\Delta v_2^{\uparrow} + \Delta v_2^{\downarrow})(1-R^2)(\mathrm{e}^{-2d_2a} - r^2\mathrm{e}^{-2(d_1+d_2)a}) \\
& - \left(\frac{E_1^{\uparrow}}{d_1} - \Delta v_0^{\uparrow}\right)(1-r^2)\mathcal{D}^{\uparrow}\mathrm{e}^{-4d_2a}\mathrm{e}^{-2d_1b} \\
& \left. + \left(\frac{E_1^{\downarrow}}{d_2} - \Delta v_0^{\downarrow}\right)(1-r^2)(1-R^2)\mathrm{e}^{-2d_2b}\right\} \\
& + \frac{\mathrm{e}^{-2d_2(z-z_1)}}{2\,\mathrm{den}}\left\{\frac{E_1^{\uparrow}}{d_1}(1-r^2)\mathcal{C}^{\uparrow} - \frac{E_2^{\uparrow}}{d_2}\left[(1+r^2-r^2\mathrm{e}^{-2d_2a})\mathcal{C}^{\uparrow} - r^2\mathcal{D}^{\uparrow}\mathrm{e}^{-2d_2a}\right]\right. \\
& - \frac{E_1^{\downarrow}}{d_2}r^2(1-r^2)(1-R^2)\mathrm{e}^{-2(d_1+d_2)a} \\
& + \frac{E_2^{\downarrow}}{d_1}r^2(1-R^2)\left(\mathrm{e}^{-2(d_1+d_2)a} - \mathrm{e}^{-2d_2a} + r^2\mathrm{e}^{-2(d_1+d_2)a} - \mathrm{e}^{-2(2d_1+d_2)a}\right) \\
& + \Delta v_1^{\uparrow}(1-r^2)\mathcal{C}^{\uparrow} - \Delta v_1^{\downarrow}r^2(1-r^2)(1-R^2)\mathrm{e}^{-2(d_1+d_2)a} \\
& - r^2(\Delta v_2^{\uparrow} + \Delta v_2^{\downarrow})(1-R^2)(\mathrm{e}^{-2d_2a} - r^2\mathrm{e}^{-2(2d_1+d_2)a}) \\
& - \left(\frac{E_1^{\uparrow}}{d_1} - \Delta v_0^{\uparrow}\right)\mathrm{e}^{-2d_1b}(1-r^2)\mathcal{C}^{\uparrow} \\
& \left. + r^2(1-r^2)(1-R^2)\left(\frac{E_1^{\downarrow}}{d_2} - \Delta v_0^{\downarrow}\right)\mathrm{e}^{-2d_2b}\mathrm{e}^{-2(d_1+d_2)a}\right\}
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
j_{\varkappa}^{\uparrow P} = {} & \frac{E^{\uparrow}}{d_1} - \frac{\mathrm{e}^{-2d_1(z-z_2)}}{2}\left(\frac{E^{\uparrow}}{d_1} - \Delta v_{0P}^{\uparrow}\right)\mathrm{e}^{-2d_1(a+b)} \\
& - \frac{\mathrm{e}^{2d_1(z-z_2)}}{2}\left[\frac{E^{\uparrow}}{d_1}(1+R^2) - \frac{E^{\downarrow}}{d_2}(1-R^2) - (\Delta v_2^{\uparrow P} + \Delta v_2^{\downarrow P})(1-R^2)\right. \\
& \left. + (1-R^2)\left(\frac{E^{\downarrow}}{d_2} - \Delta v_{0P}^{\downarrow}\right)\mathrm{e}^{-2d_2(a+b)} + R^2\left(\frac{E^{\uparrow}}{d_1} - \Delta v_{0P}^{\uparrow}\right)\mathrm{e}^{-2d_1(a+b)}\right]
\end{aligned}
\tag{B.4}
$$

where

$$
\begin{aligned}
\mathcal{C}^{\uparrow} &= 1 + r^2R^2\mathrm{e}^{-4d_1a} - 2rR\mathrm{e}^{-2d_1a}\cos 2c_1a \\
\mathcal{D}^{\uparrow} &= R^2 + r^2\mathrm{e}^{-4d_1a} - 2rR\mathrm{e}^{-2d_1a}\cos 2c_1a \\
\mathrm{den} &= 1 + 2r^2R^2\mathrm{e}^{-2(d_1+d_2)a}\cosh 2(d_1-d_2)a + r^4\mathrm{e}^{-4(d_1+d_2)a} \\
& \quad - 2rR\left[(\mathrm{e}^{-2d_1a} - r^2\mathrm{e}^{-2(d_1+2d_2)a})\cos 2c_1a - (\mathrm{e}^{-2d_2a} + r^2\mathrm{e}^{-2(2d_1+d_2)a})\cos 2c_1a\right] \\
& \quad - 2r^2\mathrm{e}^{-2(d_1+d_2)a}\left[R^2\cos 2(c_1+c_2)a + \cos 2(c_1-c_2)a\right]
\end{aligned}
$$

and Δv_0^σ, Δv_{0P}^σ are the spin-dependent voltage drops at the left contact for AP and P configurations, respectively.

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